GENERALIZED FUNCTIONAL ROUND-OFF ERROR ANALYSIS

by

J.W. Carr III
Assistant Professor of Mathematics,
University of Michigan

Generalized error bounds for (a) implicit iteration or functional solutions and (b) explicit iterations are obtained for the round-off error generated during the course of a computation.

(a) An extension is made of the work of Collatz in applying classical functional analysis to determine truncation error bounds for functional iteration, to include round-off error at each stage. This is done for a general system of non-linear functions of an arbitrary number of variables, using arbitrary norms; conditions are found to guarantee stability of the solution under round-off, and upper limits for the total error obtained.

(b) For explicit iteration, a similar round-off error analysis provides error bounds for a general system of non-linear equations, for arbitrary norms. In both cases if the norm of the iteration operator is bounded less than one, the total round-off error is bounded independent of the number of iterations.

Particular applications of the theory include the Jacobi diagonalization procedure, previously analyzed in an unpublished work of Goldstine, the Gauss Elimination Solution Methods and various techniques of numerical integration of ordinary and partial differential equations.

(In the event, Professor Carr was unable to get to the Conference owing to difficulties with transport, and hence the above paper was not presented and is not available for publication. - Ed.)